PG – 651

III Semester M.Sc. Degree Examination, December 2016
(Semester Scheme)
(RNS) (2K11 Scheme) (Repeaters)
MATHEMATICS
M302 : Numerical Analysis and Matlab Programming – I

Time : 3 Hours Max. Marks : 60

Instruction : *Answer **any five** questions by choosing **at least one** from **each** Part.*

PART – A

1. a) Derive the Newton Raphson method to solve the Non linear equation f(x) = 0, and hence obtain the rate and condition of convergence for this method.

   b) Explain Ramanujan's method to solve the non linear equation f(x) = 0, where f(x) is of the form $f(x) = 1 - (a_1 x + a_2 x^2 + a_3 x^3 + ...)$ and hence apply this method to find the smallest root of the equation $xe^x = 1$.

   c) In the year 1225, Leonardo of pisa studied the equation $f(x) = x^3 + 2x^2 + 10x - 20 = 0$ and produced the solution x = 1.368808107. Nobody knows how. Solve this equation by

      i) Linear method and

      ii) Newton Raphson method. **(4+4+4)**

2. a) Derive the Newton Raphson method to solve a system of 'n' Nonlinear equations in 'n' variables.

   b) Solve the following system on nonlinear equations

   $x_1^2 + x_2 - 37 = 0$

   $x_1 + x_2^2 - 5 = 0$

   $x_1 + x_2 + x_3 - 3 = 0$

   by the Newton raphson method derived in part a) above. **(4+8)**

3. a) Solve the following system using

$2x_1 - x_2 + x_3 = -1$

$3x_1 + 3x_2 + 9x_3 = 0$

$3x_1 + 3x_2 + 5x_3 = 4$

b) Define an $(n \times n)$ tridiagonal system of linear equations and develop the Thomas recursion algorithm.

c) Solve the following system AX = B, where

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 3 & -1 & 0 \\ 0 & 4 & 2 & 3 \\ 0 & 0 & 2 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 5 \\ 5 \\ 11 \\ 10 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

by using Thomas recursion algorithm. **(4+4+4)**

PART – B

4. a) Derive the Lagrange interpolation formula for the data values

$(x_i, f(x_i), i = 0, 1, 2....n)$ **5**

b) Let P(x) be the n^{th} degree Lagrange interpolating polynomial and the function $f(x) \in C^{n+1} [x_0, x_n]$, $x_0 < x_1 ... < x_{n-1} < x_n$ and x_i are distinct then show that

$f(x) = p(x) + E_n(x)$, where $E_n(x) = (x - x_0)(x - x_1) .... (x - x_n) f^{n+1}(\xi) / (n+1)!$. **4**

c) Take $f(x) = e^x$ on [0, 1]. Determine the following :

i) $| e^x - P_1(x) |$ and

ii) $| e^x - P_2(x) |$

where $P_1(x)$ and $P_2(x)$ are linear and quadratic interpolations on [0, 1] with $x \in [x_0, x_1]$ and $x \in [x_0, x_2]$, $x_i = x_0 + ih$ (i = 0, 1, 2) respectively. **3**

5. a) Derive the Hermite interpolation function H(x) that interpolates the data

$((x_i, f(x_i), f'(x_i)), i = 0, 1, 2, ...,n)$, when $f \in C^{2n+2} [a, b]$ and

$a = x_0 < x_1 < x_2 < ... < x_{n-1} < x_n = b$. **5**

b) Obtain rational function approximation of the form

$$\frac{a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5}{1 + b_1x + b_2x^2 + b_3x^3 + b_4x^4 + b_5x^5}$$

for $(\tan^{-1} x) \approx x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \frac{1}{9}x^9$. **4**

c) Fit cubic splines $s_i(x)$, i = 1, 2, 3 with zero second derivative end conditions and passing through the data points (–2, 4), (–1, –1), (0, 2), (1, 1) and (2,8). **3**

6. a) Let $(x_i^{(n)}, i = 1, 2, ..., n)$ be the roots of the n^{th} degree Legendre polynomial $P_n(x)$

then show that $\int_{-1}^{1} P(x)\,dx = \sum_{i=1}^{n} C_i^{(n)} \; P(x_i^{(n)}),\; C_i^{(n)} = \int_{-1}^{1}\left(\prod_{j=1,\, j\neq i}^{n}\left(\frac{x - x_j^{(n)}}{x_i^{(n)} - x_j^{(n)}}\right)\right) dx$

where P(x) is a polynomial of degree $\leq (2n)$. **5**

b) Prove that $C_i^{(n)} = \frac{2}{(1-(x_i^{(n)})^2)[P_n'(x_i^{(n)})]^2}$ **4**

c) Compute the values $\left(\left(C_i^{(n)}, x_i^{(n)}\right), i = 1, 2, ..., n\right)$ for n = 2, 3, 4, 5. **3**

PART – C

7. a) Discus about the variables and different types of their initialization. **6**

b) Write a matlab/scilab program to find the palindrome of the given number. **6**

8. a) Explain the concept of decision making statements with suitable examples. **6**

b) Discuss the concept of functions. Write a matlab/scilab program to evaluate the definite integral using Simpson's $1/3$ rule. **6**